State-of-the-art SAT Solving

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Satisfiability (SAT) Solving Has Many Applications

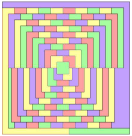
formal verification

security

bioinformatics

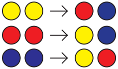
train safety

planning and scheduling



automated theorem proving

exploit generation



term rewriting termination

encode → **SAT solver** → **decode**

Dress Code as Satisfiability Problem

The SAT problem: Can a formula in propositional logic be satisfied?

Propositional logic

- Boolean variables : **tie** and **shirt** (for the example below)
- Logic symbols : $\neg$ (not), $\vee$ (or), $\wedge$ (and)
- Literals : **tie**, $\neg$**tie**, **shirt**, and $\neg$**shirt**

Three conditions / clauses :

- not wearing a **tie** nor a **shirt** is impolite $(\textbf{tie} \vee \textbf{shirt})$
- clearly one should not wear a **tie** without a **shirt** $(\neg\textbf{tie} \vee \textbf{shirt})$
- wearing a **tie** and a **shirt** is overkill $\neg(\textbf{tie} \wedge \textbf{shirt}) \equiv (\neg\textbf{tie} \vee \neg\textbf{shirt})$

Is the formula $(\textbf{tie} \vee \textbf{shirt}) \wedge (\neg\textbf{tie} \vee \textbf{shirt}) \wedge (\neg\textbf{tie} \vee \neg\textbf{shirt})$ satisfiable?

A Larger, but Still Small Satisfiability Problem

Is the formula below satisfiable?

$$
\begin{aligned}
&(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_1 \vee x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_3 \vee \neg x_4) \wedge \\
&(x_1 \vee x_4 \vee x_5) \wedge (\neg x_1 \vee \neg x_4 \vee \neg x_5) \wedge (x_2 \vee x_3 \vee x_5) \wedge (\neg x_2 \vee \neg x_3 \vee \neg x_5) \wedge \\
&(x_1 \vee x_5 \vee x_6) \wedge (\neg x_1 \vee \neg x_5 \vee \neg x_6) \wedge (x_2 \vee x_4 \vee x_6) \wedge (\neg x_2 \vee \neg x_4 \vee \neg x_6) \wedge \\
&(x_1 \vee x_6 \vee x_7) \wedge (\neg x_1 \vee \neg x_6 \vee \neg x_7) \wedge (x_2 \vee x_5 \vee x_7) \wedge (\neg x_2 \vee \neg x_5 \vee \neg x_7) \wedge \\
&(x_3 \vee x_4 \vee x_7) \wedge (\neg x_3 \vee \neg x_4 \vee \neg x_7) \wedge (x_1 \vee x_7 \vee x_8) \wedge (\neg x_1 \vee \neg x_7 \vee \neg x_8) \wedge \\
&(x_2 \vee x_6 \vee x_8) \wedge (\neg x_2 \vee \neg x_6 \vee \neg x_8) \wedge (x_3 \vee x_5 \vee x_8) \wedge (\neg x_3 \vee \neg x_5 \vee \neg x_8) \wedge \\
&(x_1 \vee x_8 \vee x_9) \wedge (\neg x_1 \vee \neg x_8 \vee \neg x_9) \wedge (x_2 \vee x_7 \vee x_9) \wedge (\neg x_2 \vee \neg x_7 \vee \neg x_9) \wedge \\
&\qquad (\neg x_3 \vee \neg x_6 \vee \neg x_9) \wedge (x_4 \vee x_5 \vee x_9) \wedge (\neg x_4 \vee \neg x_5 \vee \neg x_9)
\end{aligned}
$$

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$$
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&(x_1 \vee x_4 \vee x_5) \wedge (\neg x_1 \vee \neg x_4 \vee \neg x_5) \wedge (x_2 \vee x_3 \vee x_5) \wedge (\neg x_2 \vee \neg x_3 \vee \neg x_5) \wedge \\
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&(x_1 \vee x_6 \vee x_7) \wedge (\neg x_1 \vee \neg x_6 \vee \neg x_7) \wedge (x_2 \vee x_5 \vee x_7) \wedge (\neg x_2 \vee \neg x_5 \vee \neg x_7) \wedge \\
&(x_3 \vee x_4 \vee x_7) \wedge (\neg x_3 \vee \neg x_4 \vee \neg x_7) \wedge (x_1 \vee x_7 \vee x_8) \wedge (\neg x_1 \vee \neg x_7 \vee \neg x_8) \wedge \\
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&\qquad (\neg x_3 \vee \neg x_6 \vee \neg x_9) \wedge (x_4 \vee x_5 \vee x_9) \wedge (\neg x_4 \vee \neg x_5 \vee \neg x_9)
\end{aligned}
$$

Yes. The correctness of the solution is easy to check.

A Larger, but Still Small Satisfiability Problem

Is the formula below still satisfiable?

$$
\begin{aligned}
&(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor \neg x_3) \land (x_1 \lor x_3 \lor x_4) \land (\neg x_1 \lor \neg x_3 \lor \neg x_4) \land \\
&(x_1 \lor x_4 \lor x_5) \land (\neg x_1 \lor \neg x_4 \lor \neg x_5) \land (x_2 \lor x_3 \lor x_5) \land (\neg x_2 \lor \neg x_3 \lor \neg x_5) \land \\
&(x_1 \lor x_5 \lor x_6) \land (\neg x_1 \lor \neg x_5 \lor \neg x_6) \land (x_2 \lor x_4 \lor x_6) \land (\neg x_2 \lor \neg x_4 \lor \neg x_6) \land \\
&(x_1 \lor x_6 \lor x_7) \land (\neg x_1 \lor \neg x_6 \lor \neg x_7) \land (x_2 \lor x_5 \lor x_7) \land (\neg x_2 \lor \neg x_5 \lor \neg x_7) \land \\
&(x_3 \lor x_4 \lor x_7) \land (\neg x_3 \lor \neg x_4 \lor \neg x_7) \land (x_1 \lor x_7 \lor x_8) \land (\neg x_1 \lor \neg x_7 \lor \neg x_8) \land \\
&(x_2 \lor x_6 \lor x_8) \land (\neg x_2 \lor \neg x_6 \lor \neg x_8) \land (x_3 \lor x_5 \lor x_8) \land (\neg x_3 \lor \neg x_5 \lor \neg x_8) \land \\
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&(x_3 \lor x_6 \lor x_9) \land (\neg x_3 \lor \neg x_6 \lor \neg x_9) \land (x_4 \lor x_5 \lor x_9) \land (\neg x_4 \lor \neg x_5 \lor \neg x_9)
\end{aligned}
$$

No. Adding a single clause eliminates all solutions.

Checking a No answer can be expensive.

Satisfiability as the Cornerstone of the P = NP Question

A fundamental question in computer science asks whether searching for a solution is harder than verifying a given solution.

For example, consider the Sudoku on the right: Is searching for the solution harder than verifying a given candidate solution?

	4		3					
						7	9	
			6					
			1	4		5		
9							1	
2								6
				7	2			
	5					8		
				9				

Satisfiability as the Cornerstone of the P = NP Question

A fundamental question in computer science asks whether searching for a solution is harder than verifying a given solution.

For example, consider the Sudoku on the right: Is searching for the solution harder than verifying a given candidate solution?

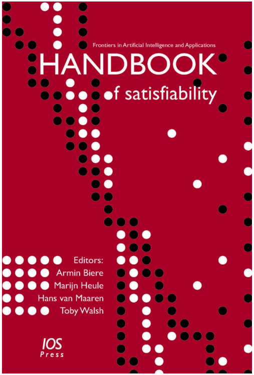
1	4	7	3	8	9	2	6	5
5	8	6	2	1	4	7	9	3
3	9	2	6	5	7	1	8	4
8	7	3	1	4	6	5	2	9
9	6	4	7	2	5	3	1	8
2	1	5	9	3	8	4	7	6
6	3	8	5	7	2	9	4	1
7	5	9	4	6	1	8	3	2
4	2	1	8	9	3	6	5	7

This is the P = NP question. Solving it is worth $1,000,000.

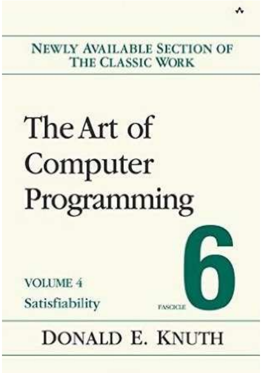
Cook-Levin Theorem [1971]: SAT is NP-complete.
Searching is as easy as verifying if and only if this holds for SAT.

Enormous Progress in the Last Two Decades

mid '90s: formulas solvable with thousands of variables and clauses
now: formulas solvable with millions of variables and clauses



Edmund Clarke: *"a key technology of the 21st century"*



Donald Knuth: *"evidently a killer app, because it is key to the solution of so many other problems"*

Overview

Search for Lemmas (now)

- Learning Lemmas
- Data-structures
- Heuristics

Search for Simplification (after the break)

- Variable elimination
- Blocked clause elimination
- Unhiding redundancy

Conflict-driven SAT solvers: Search and Analysis

$$(x_1 \lor x_4) \land$$
$$(x_3 \lor \neg x_4 \lor \neg x_5) \land$$
$$(\neg x_3 \lor \neg x_2 \lor \neg x_4) \land$$
$$\mathcal{F}_{\text{extra}}$$

Conflict-driven SAT solvers: Search and Analysis

$$
\begin{aligned}
&(x_1 \lor x_4) \land \\
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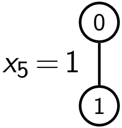
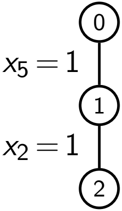


Conflict-driven SAT solvers: Search and Analysis

$$(x_1 \vee x_4) \wedge$$
$$(x_3 \vee \neg x_4 \vee \neg x_5) \wedge$$
$$(\neg x_3 \vee \neg x_2 \vee \neg x_4) \wedge$$
$$\mathcal{F}_{\text{extra}}$$



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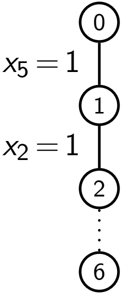

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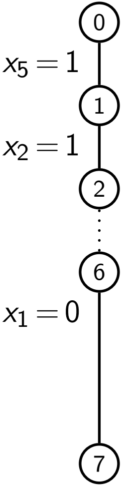

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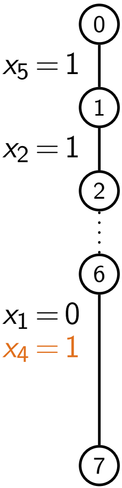

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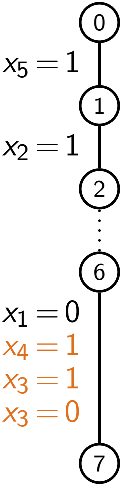

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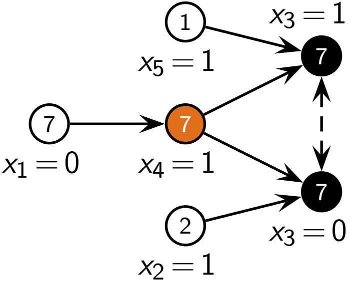

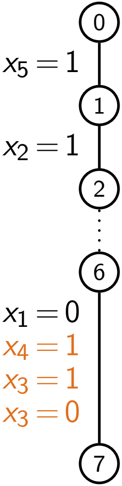

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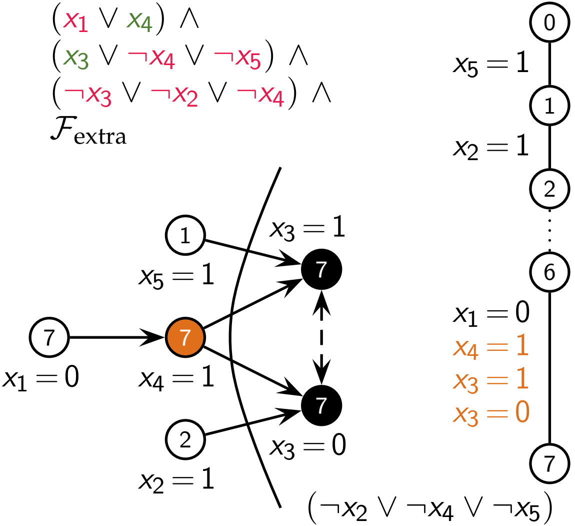

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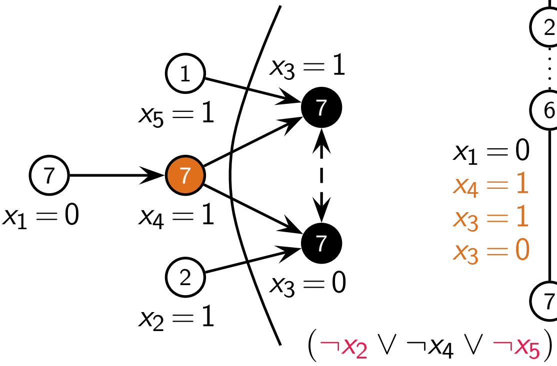

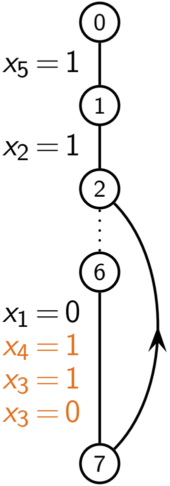

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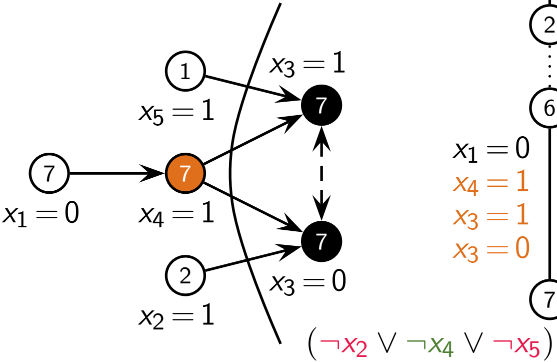

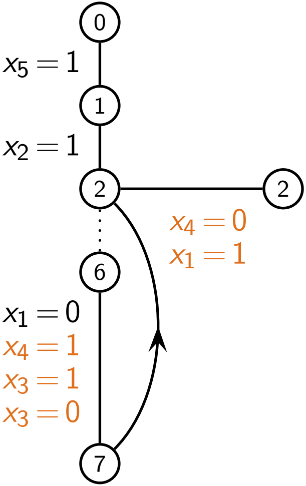

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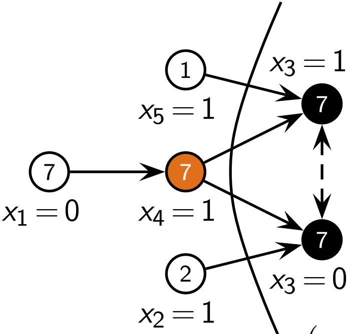
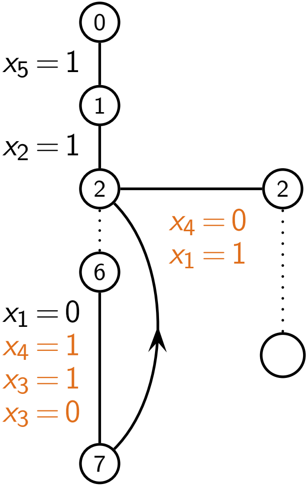


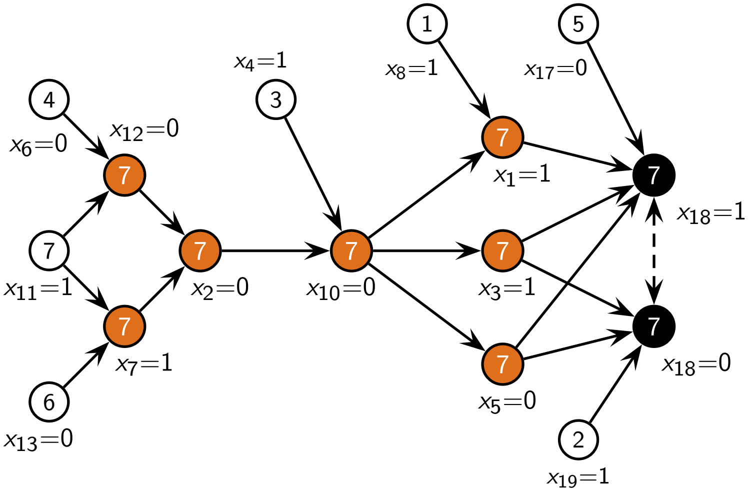
$(\neg x_2 \lor \neg x_4 \lor \neg x_5)$



Conflict-driven SAT solvers: Pseudo-code

while TRUE **do**
    $l_{\text{decision}} := \textsc{GetDecisionLiteral}(\ )$
    If no l_{decision} **then return** `satisfiable`
    $\mathcal{F} := \textsc{Simplify}(\ \mathcal{F}(l_{\text{decision}} \leftarrow 1)\ )$
    while $\mathcal{F}$ contains $C_{\text{falsified}}$ **do**
        $C_{\text{conflict}} := \textsc{AnalyzeConflict}(\ C_{\text{falsified}}\ )$
        If $C_{\text{conflict}} = \emptyset$ **then return** `unsatisfiable`
        $\textsc{BackTrack}(\ C_{\text{conflict}}\ )$
        $\mathcal{F} := \textsc{Simplify}(\ \mathcal{F} \cup \{C_{\text{conflict}}\}\ )$
    end while
end while

Learning conflict clauses [Marques-SilvaSakallah'96]



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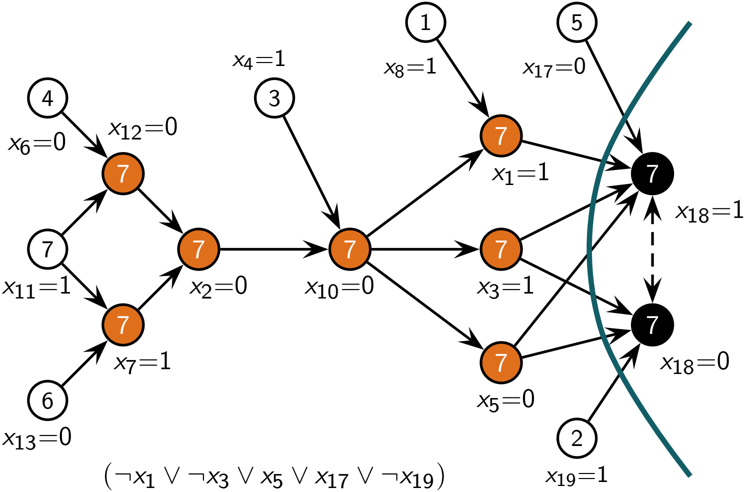
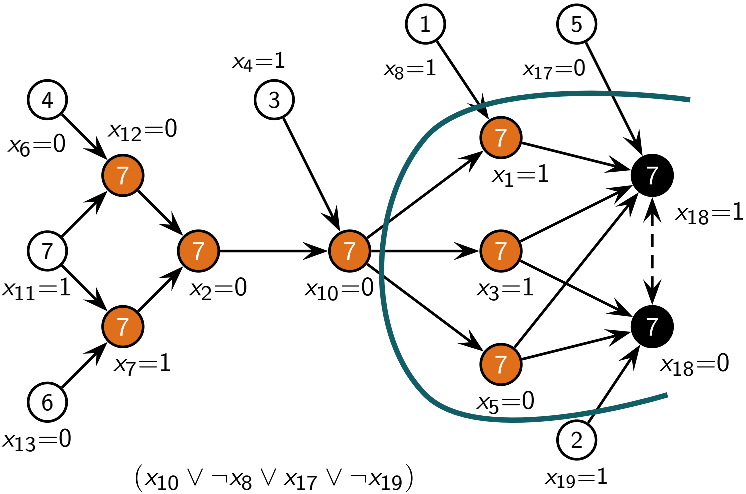


$(\neg x_1 \vee \neg x_3 \vee x_5 \vee x_{17} \vee \neg x_{19})$

tri-asserting clause

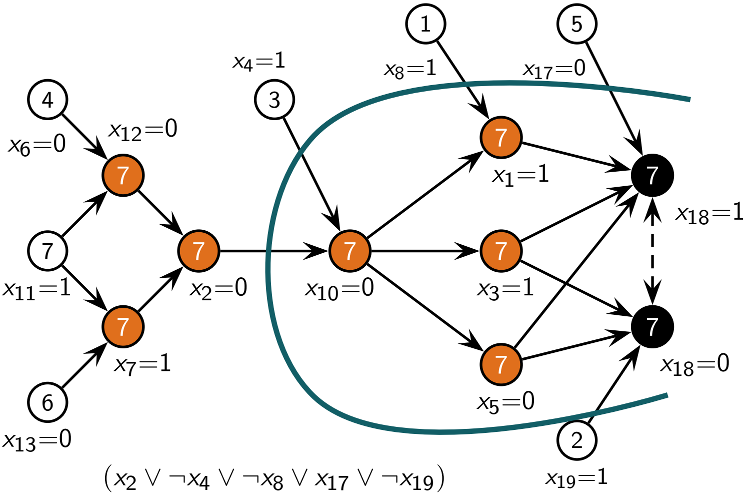
Learning conflict clauses [Marques-SilvaSakallah'96]



$(x_{10} \vee \neg x_8 \vee x_{17} \vee \neg x_{19})$

first unique implication point

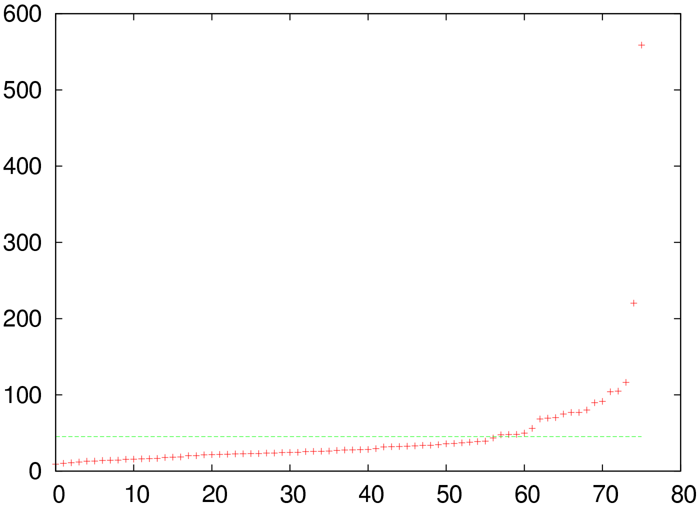
Learning conflict clauses [Marques-SilvaSakallah'96]



$(x_2 \vee \neg x_4 \vee \neg x_8 \vee x_{17} \vee \neg x_{19})$

second unique implication point

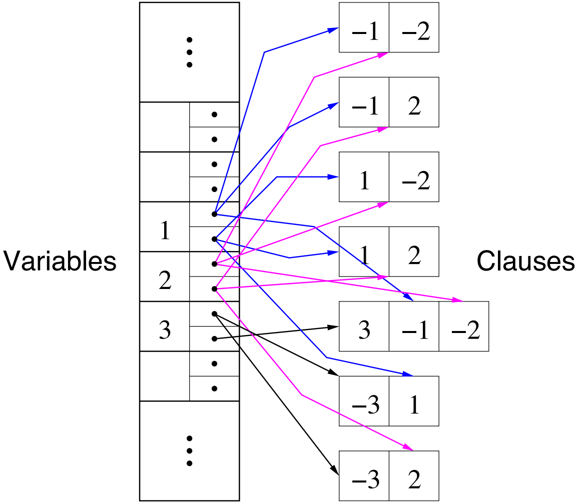
Average Learned Clause Length



Data-structures

Watch pointers

Simple data structure for unit propagation



Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\varphi = \{x_1 = *, x_2 = *, x_3 = *, x_4 = *, x_5 = *, x_6 = *\}$$

$\neg x_1$	x_2	$\neg x_3$	$\neg x_5$	x_6

x_1	$\neg x_3$	x_4	$\neg x_5$	$\neg x_6$

Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\varphi = \{x_1 = *, x_2 = *, x_3 = *, x_4 = *, x_5 = \mathbf{1}, x_6 = *\}$$

$\neg x_1$	x_2	$\neg x_3$	$\neg x_5$	x_6

x_1	$\neg x_3$	x_4	$\neg x_5$	$\neg x_6$

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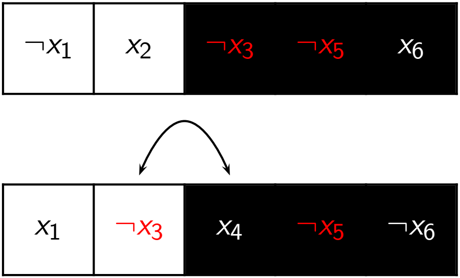

Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\varphi = \{x_1 = *, x_2 = *, x_3 = 1, x_4 = *, x_5 = 1, x_6 = *\}$$

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x_1	x_4	$\neg x_3$	$\neg x_5$	$\neg x_6$

Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

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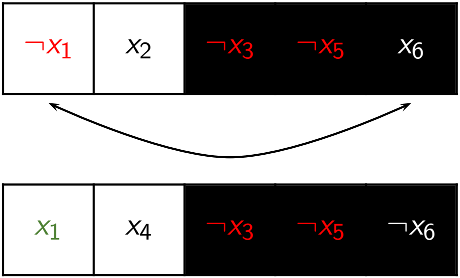

Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\varphi = \{x_1 = 1, x_2 = *, x_3 = 1, x_4 = *, x_5 = 1, x_6 = *\}$$

x_6	x_2	$\neg x_3$	$\neg x_5$	$\neg x_1$

x_1	x_4	$\neg x_3$	$\neg x_5$	$\neg x_6$

Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\varphi = \{x_1 = 1, x_2 = *, x_3 = 1, x_4 = \mathbf{0}, x_5 = 1, x_6 = *\}$$

x_6	x_2	$\neg x_3$	$\neg x_5$	$\neg x_1$

x_1	x_4	$\neg x_3$	$\neg x_5$	$\neg x_6$

Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\varphi = \{x_1 = 1, x_2 = \mathbf{0}, x_3 = 1, x_4 = 0, x_5 = 1, x_6 = *\}$$

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Conflict-driven: Watch pointers (2) [MoskewiczMZZM'01]

Only examine (get in the cache) a clause when both

- a watch pointer gets falsified
- the other one is not satisfied

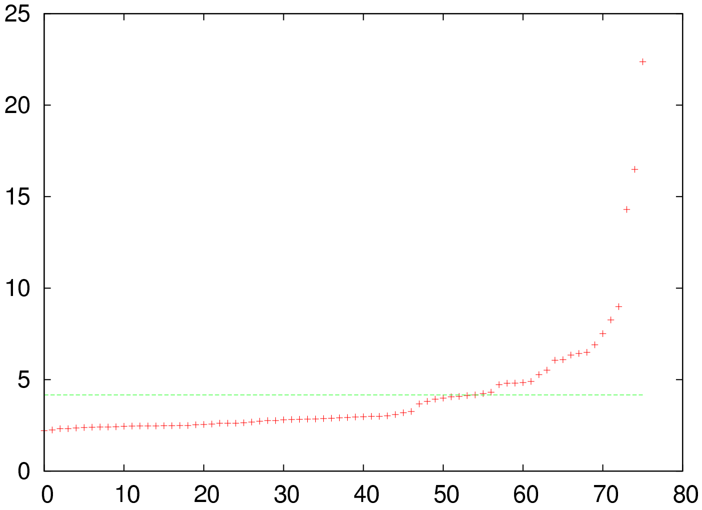
While backjumping, just unassign variables

Conflict clauses $\rightarrow$ watch pointers

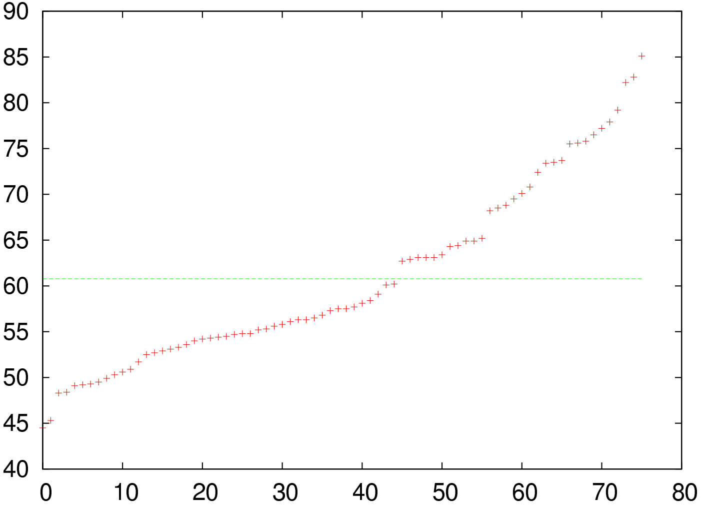
No detailed information available

Not used for binary clauses

Average Number Clauses Visited Per Propagation



Percentage visited clauses with other watched literal true



Heuristics

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Restart strategies

- aim: avoid heavy-tail behavior [GomesSelmanCrato'97]
- plus: focus search on recent conflicts when combined with dynamic heuristics

Variable selection heuristics

Based on the occurrences in the (reduced) formula

- examples: Jeroslow-Wang, Maximal Occurrence in clauses of Minimal Size (MOMS), look-aheads
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Variable State Independent Decaying Sum (VSIDS)

- original idea (zChaff): for each conflict, increase the score of involved variables by 1, half all scores each 256 conflicts [MoskewiczMZZM'01]
- improvement (MiniSAT): for each conflict, increase the score of involved variables by δ and increase $\delta := 1.05\delta$ [EenSörensson'03]

Visualization of VSIDS in PicoSAT

http://www.youtube.com/watch?v=MOjhFywLre8

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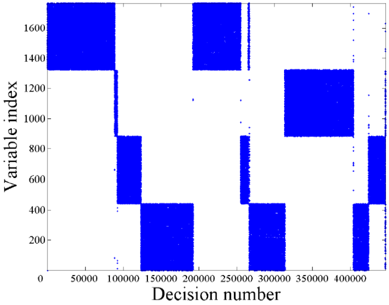
- negative branching (early MiniSAT) [EenSörensson'03]

Based on the last implied value (phase-saving)

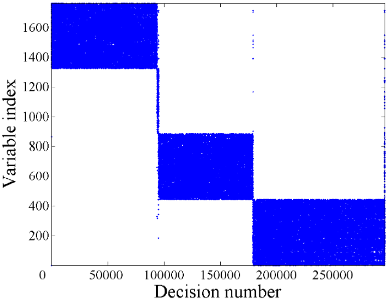
- introduced to CDCL [PipatsrisawatDarwiche'07]
- already used in local search [HirschKojevnikov'01]

Heuristics: Phase-saving [PipatsrisawatDarwiche'07]

Selecting the last implied value remembers solved components



negative branching



phase-saving

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Restarts in CDCL solvers:

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- Geometrical restart: e.g. 100, 150, 225, 333, 500, 750, . . .
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Rapid restarts by reusing trail:
[vanderTakHeuleRamos'11]

- Partial restart same effect as full restart
- Optimal strategy Luby-1: 1, 1, 2, 1, 1, 2, 4, . . .

Conflict-Clause Minimization

Self-Subsumption

Use self-subsumption to shorten conflict clauses

$$\frac{C \vee l \quad D \vee \neg l}{D} \quad C \subseteq D \qquad \frac{(a \vee b \vee l) \quad (a \vee b \vee c \vee \neg l)}{(a \vee b \vee c)}$$

Conflict clause minimization is an important optimization.

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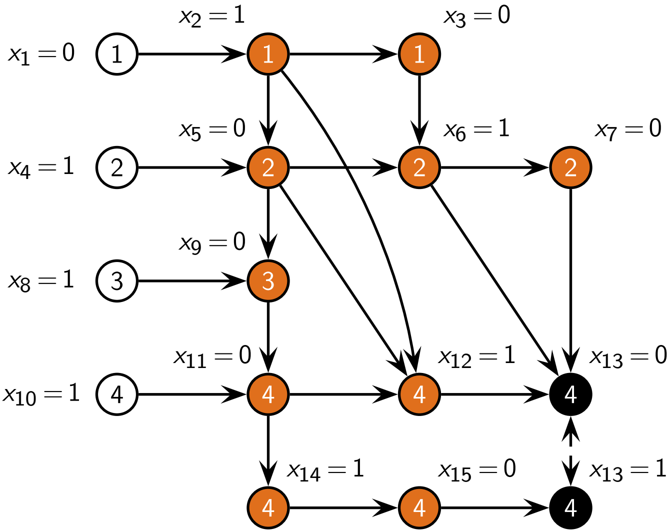
$$\frac{C \vee l \quad D \vee \neg l}{D} \quad C \subseteq D \qquad \frac{(a \vee b \vee l) \quad (a \vee b \vee c \vee \neg l)}{(a \vee b \vee c)}$$

Conflict clause minimization is an important optimization.

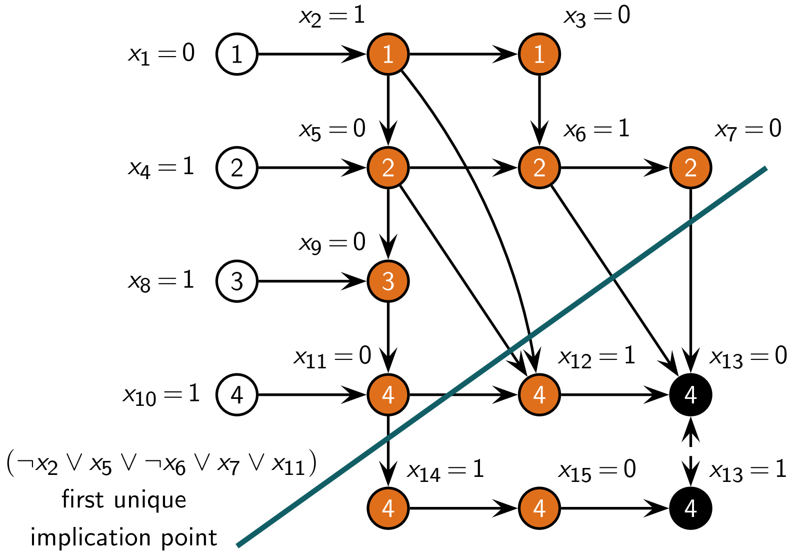
Use implication chains to further minimization:

$$\ldots(\neg a \vee b)(\neg b \vee c)(a \vee c \vee d)\ldots \quad \Rightarrow$$
$$\ldots(\neg a \vee b)(\neg b \vee c)(c \vee d)\ldots$$

Conflict-clause minimization [SörenssonBiere'09]



Conflict-clause minimization [SörenssonBiere'09]



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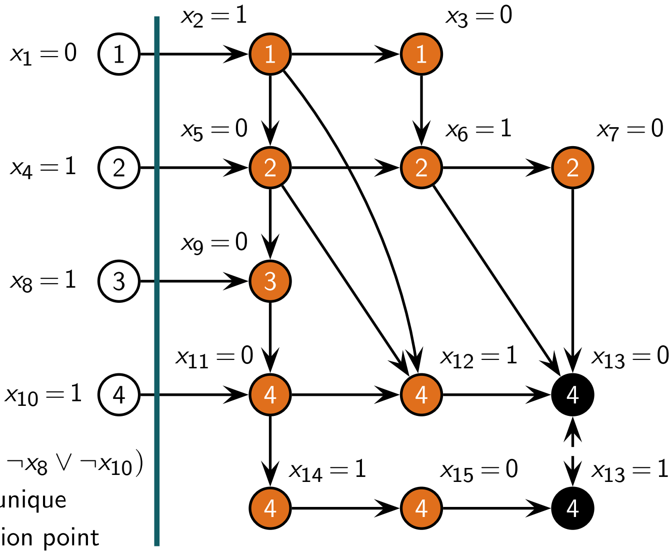
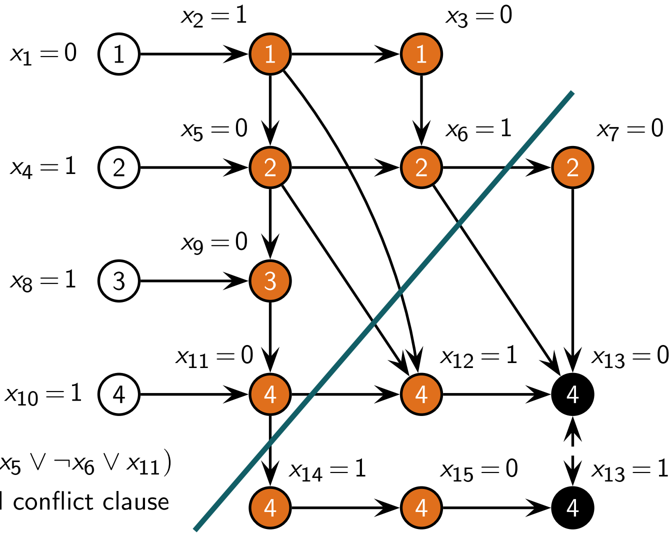


$(x_1 \vee \neg x_4 \vee \neg x_8 \vee \neg x_{10})$

last unique

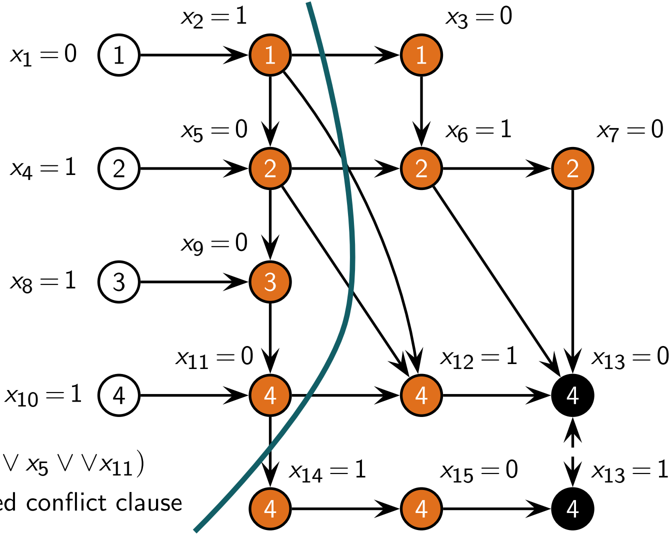
implication point

Conflict-clause minimization [SörenssonBiere'09]



$(\neg x_2 \vee x_5 \vee \neg x_6 \vee x_{11})$
reduced conflict clause

Conflict-clause minimization [SörenssonBiere'09]



$(\neg x_2 \vee x_5 \vee \vee x_{11})$
minimized conflict clause

Conclusions: state-of-the-art CDCL solver

Key contributions to CDCL solvers:

- concept of conflict clauses (grasp) [Marques-SilvaSakallah'96]
- restart strategies [GomesSC'97,LubySZ'93]
- 2-watch pointers and VSIDS (zChaff) [MoskewiczMZZM'01]
- efficient implementation (Minisat) [EenSörensson'03]
- phase-saving (Rsat) [PipatsrisawatDarwiche'07]
- conflict-clause minimization [SörenssonBiere'09]

+ Pre- and in-processing techniques

State-of-the-art SAT Solving

Marijn J.H. Heule

THE UNIVERSITY OF TEXAS AT AUSTIN

SC^2 Summer School, July 31, 2017