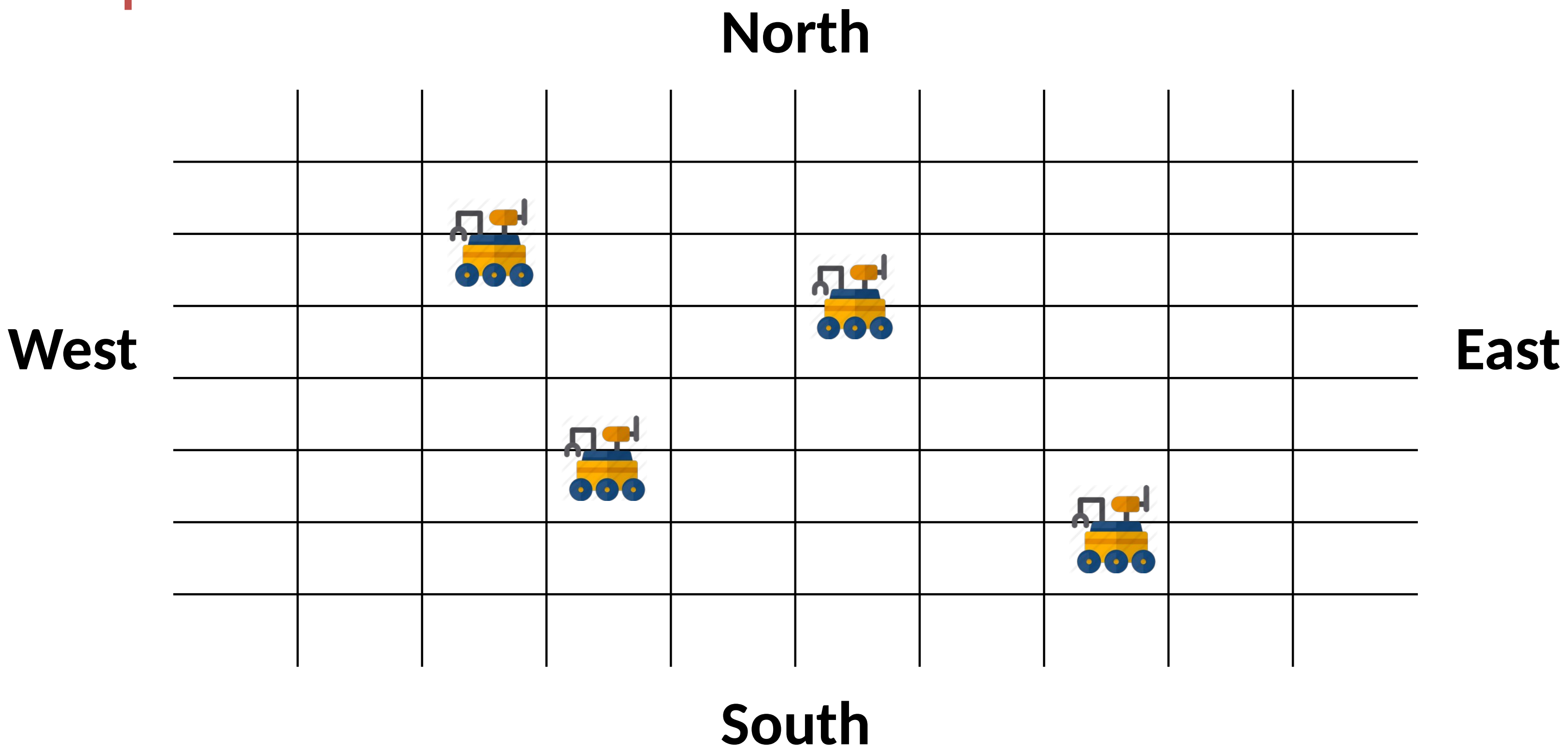
CS:5810
Formal Methods in Software Engineering

Case Study: Autonomous Rovers

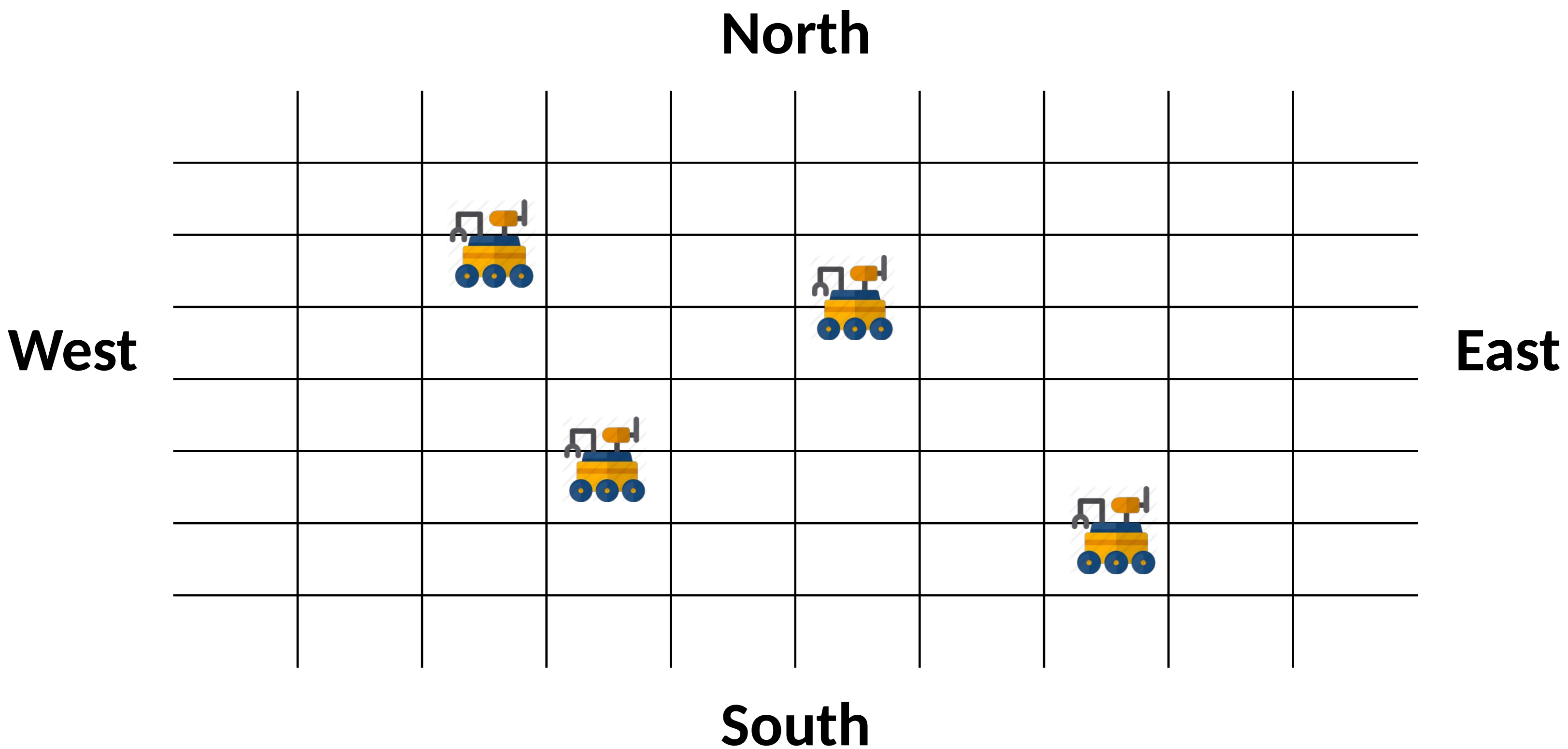
The Task

- Model in Alloy a dynamic domain involving several rovers moving on a two-dimensional space



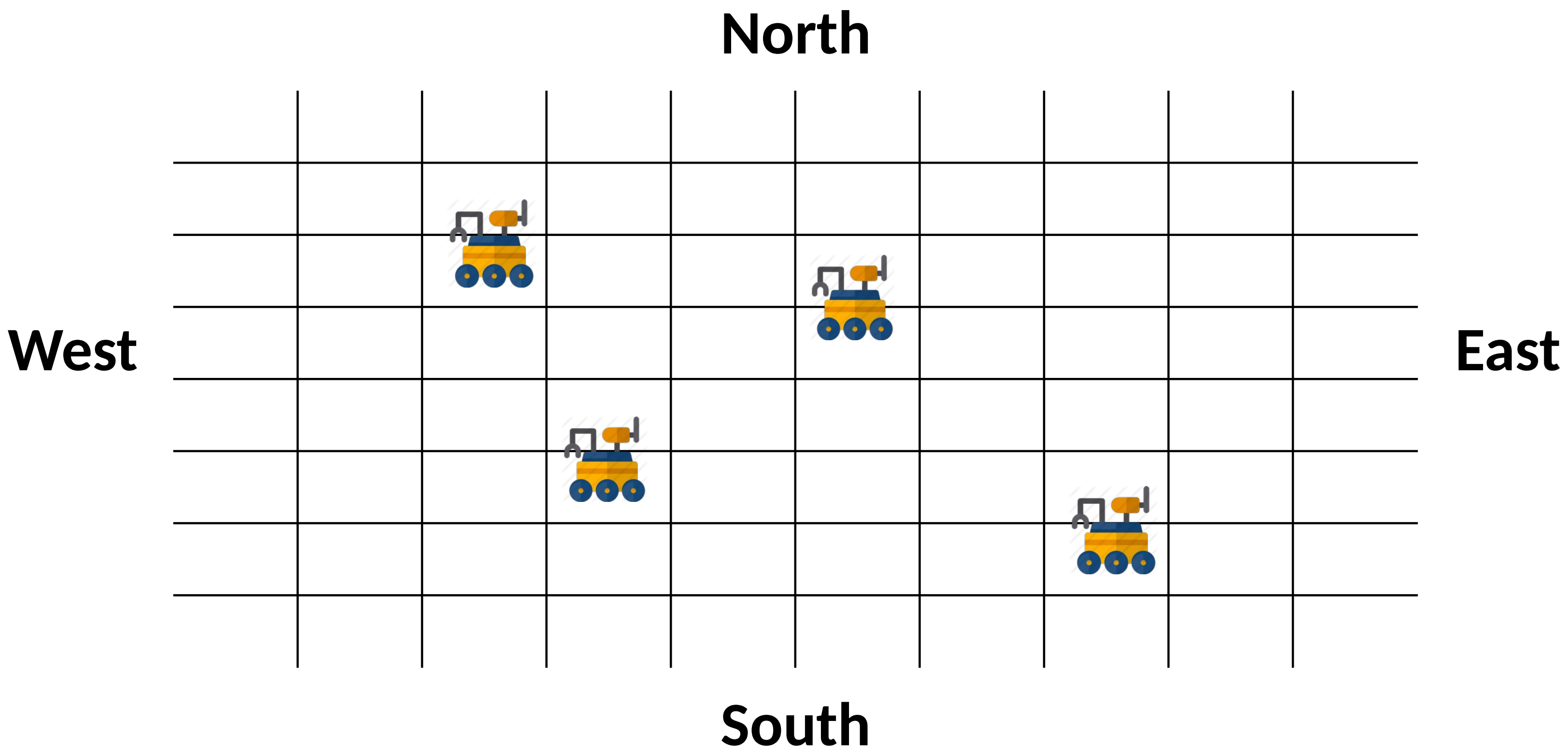
Facts about the System

- There are one or more identical rovers
- Each rover can be turned on and off



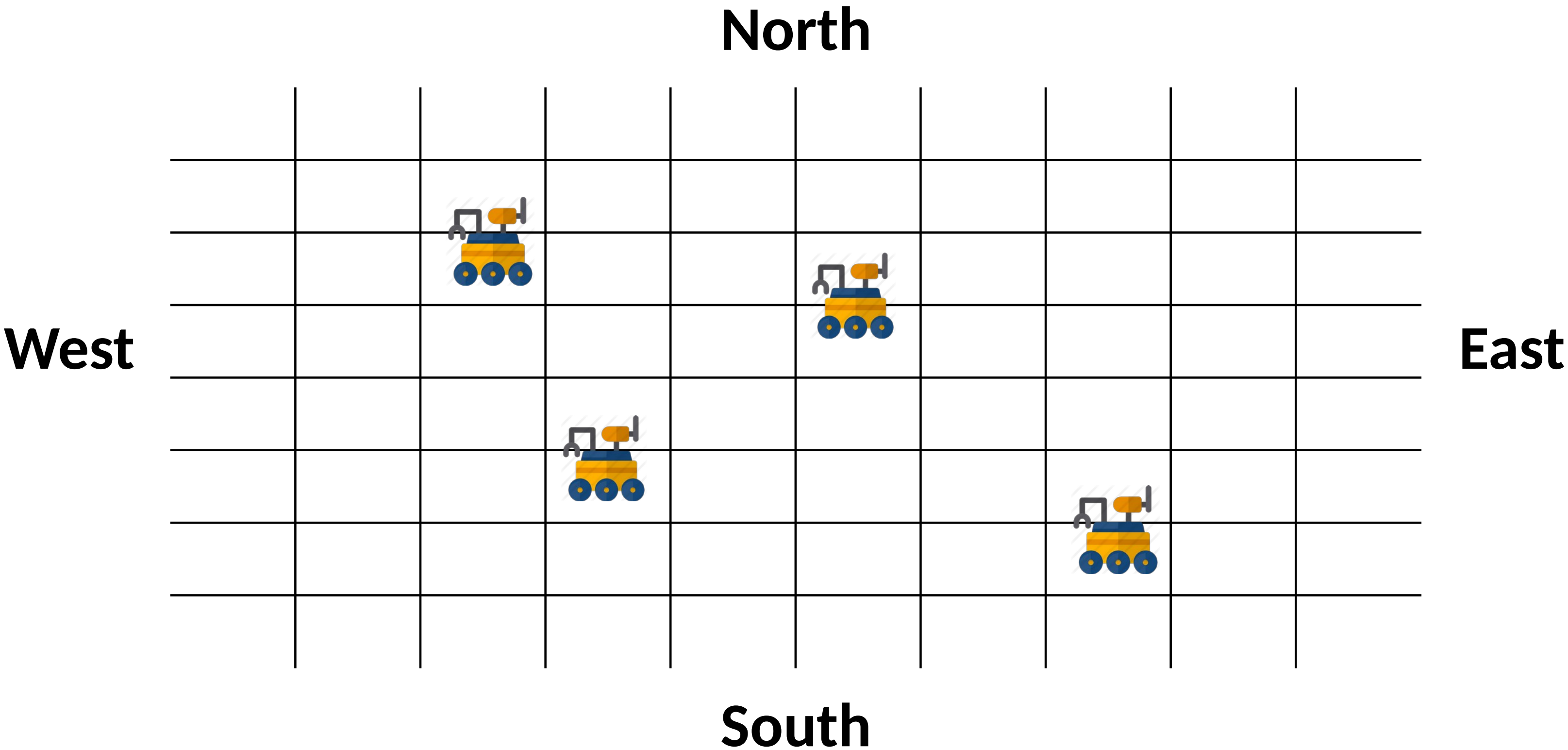
Facts about the System

- Each rover can only move forward, or turn in place to the left or to the right



Facts about the System

- We will model both static and dynamic aspects of the system



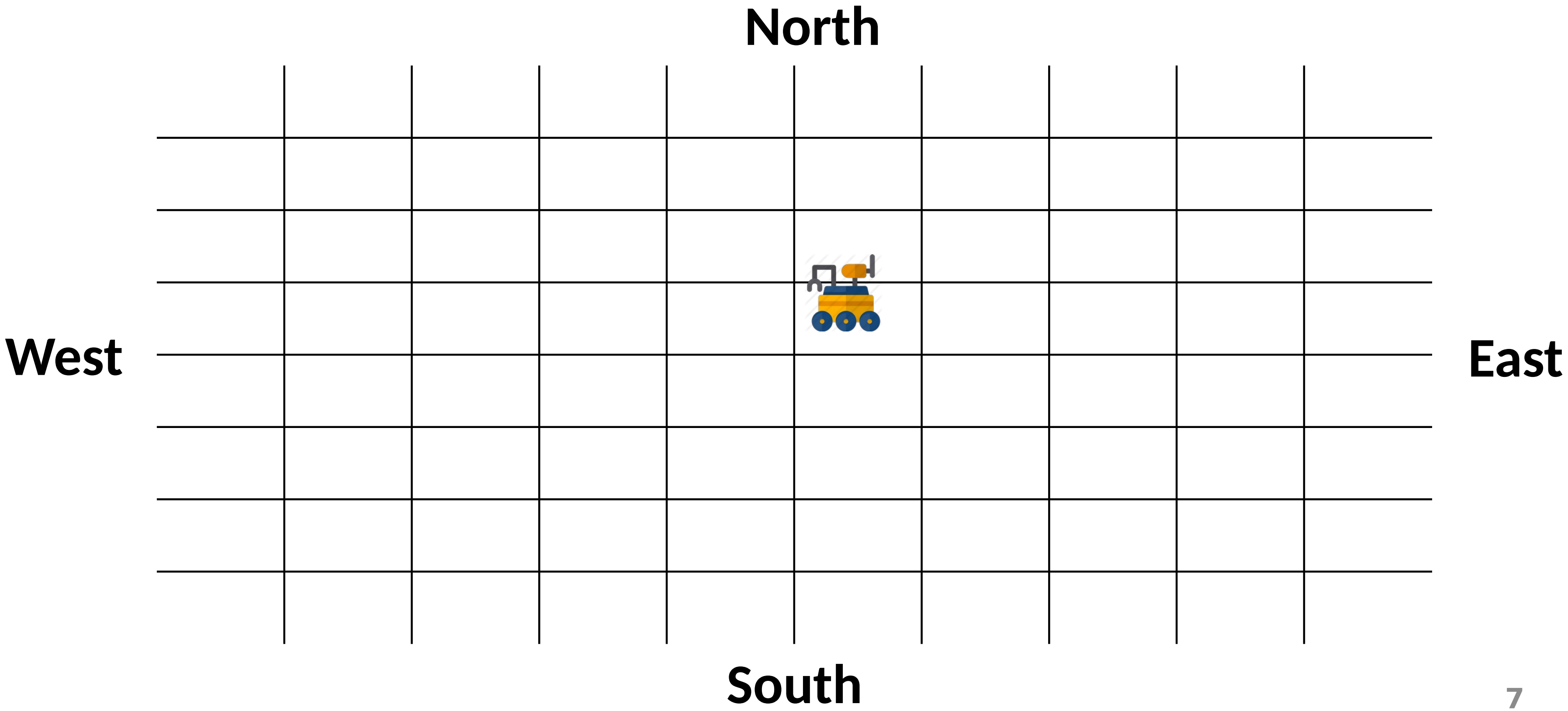
Simplifying Modeling Choices

1) We adopt an interleaving model of time: only one action is performed, by one of the rovers, at a time

2) The two dimensional space is a discrete grid, with
   - the X-coordinate growing indefinitely in the West-East direction and
   - the Y-coordinate growing indefinitely in the South-North

Simplifying Modeling Choices

3) Rovers move only by one position at a time and along the X,Y axes.



Simplifying Modeling Choices

4) A rover turns left or right by exactly 90 degrees
5) A rover can move only in the direction it is facing

Signatures and Fields

```
open util/ordering [Time] as T
open util/ordering [Coor] as C

-- Coordinates, strictly ordered
sig Time {}
sig Coor {}

-- Position models the individual positions
-- in the grid
sig Position {  x: Coor,  y: Coor}
```

Signatures and Fields

```
-- The four cardinal directions
abstract sig Direction {}

one sig North, South, East, West extends
    Direction {}
```

Signatures and Fields

```
some sig Rover {
    -- Direction rover is facing at any one time
    dir: Direction one -> Time,

    -- Rover's position at any one time
    pos: Position one -> Time,

    -- Rover's on/off status at any one time
    on: set Time
}
```

Operators

Turn on
Turn off
Turn left
Turn right
Go

Turn On Operator

```
pred turn_on [rov: Rover, t,t': Time] {
    -- Pre-condition
    Rover is off at time t (!is_on)

    -- Post-condition
    Rover is on at time t’ (is_on)

    -- Frame condition
    All other rovers stay on or off as they were (no_on_changes)
    No rover changes direction (no_direction_changes)
    No rover changes position (no_position_changes)
}
```

Turn Left Operator

```
pred turn_left [rov: Rover, t,t': Time] {
    -- Pre-condition
    Rover is on at time t (is_on)

    -- Post-condition
    Direction Changes (could be North, South, East, or West)

    -- Frame condition
    All rovers stay on or off as they were (no_on_changes)
    No other rover changes direction (no_direction_changes)
    No rover changes position (no_position_changes)
}
```

If-Then-Else in Alloy

$Expr_1$ (=>, implies) $Expr_2$ else $Expr_3$

- $Expr_1$ is a Boolean expression
- $Expr_2$ and $Expr_3$ can be either Boolean or Set expression

E.g. let parents_in_law =

```
(John.spouse = Mary => Mary.parents
 else John.spouse = Lily => Lily.parents
   else none)
```

Go Operator

```
pred go[rov: Rover, d: Direction, t,t': Time] {
    -- Pre-condition
    Rover is on at time t (is_on)
    d is rover's direction at time t

    -- Post-condition
    Position Changes (could move towards North, South, East, or
    West)
    (next_pos[p: Position, d: Direction]: Position)
    -- Frame condition
    All rovers stay on or off as they were (no_on_changes)
    No rover changes direction (no_direction_changes)
    No other rover changes position (no_position_changes)
}
```

The Module Ordering

```
// return the predecessor of e, or empty set if e is
// the first element
fun prev [e: S]: lone S { e.(Ord.Prev) }

// return the successor of e, or empty set of e is
// the last element
fun next [e: S]: lone S { e.(Ord.Next) }
```

Transition System

```
pred System {
    init[T/first]
    all t: Time – T/last | transitions[t, T/next[t]]
}
```

- Facts

-- P0 is the origin position of the coordinate system

- Init

-- Rover R1 is at the origin position, facing East and turned off
-- The other rovers, if any, are at a different position than R1's

- Transitions

-- Some rover turn on, off, left, right, or go

System Goal

```
pred goal[t: Time]{
   -- R1 is not at the origin
   R1.pos.t != P0
   -- R1 is facing north
   R1.dir.t = North
}
pred goalCheck{
    one Rover
    System
    some t : Time | goal[t]
}
```